JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 33, No. 1, January–February 2010

Engineering Note

Homing Guidance Law for Cooperative Attack of Multiple Missiles

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DOI: 10.2514/1.40136

I. Introduction

OVER the past few years, there have been significant efforts devoted to the research and development of cooperative unmanned systems [1–3]. The formation flying of multiple unmanned aerial vehicles (UAVs) has been studied for radar deception, reconnaissance, surveillance, and surface-to-air-missile jamming in military operations. An example of a cooperative operational scenario of multiple vehicles is that of a small UAV flying over an urban area, dispensing multiple micro aerial vehicles to examine points of interest from close distances [4]. A group of well-organized low-cost multiple vehicles can be far superior to a single high-technology and high-cost UAV in effectiveness. Tactical missile systems as well as UAVs provide more capabilities when they are organized as a coordinated group than when they are operated independently.

Modern antiship missiles need to be able to penetrate the formidable defensive systems of battleships such as antiair defense missile systems and close-in weapon system (CIWS). CIWS is a naval shipboard weapon system for detecting and destroying incoming antiship missiles and enemy aircraft at short range. These defensive weapons with powerful fire capability and various strategies seriously intimidate the survivability of the conventional antiship missiles. Hence, antiship missile developers have made great efforts to develop a high-performance missile system with ultimate sea-skimming flight and terminal evasive maneuvering capabilities despite a huge cost.

On the other hand, cooperative attack strategies have been studied to enhance survivability of the conventional ones. Here, a cooperative attack means that multiple missiles attack a single target or multiple targets cooperatively or, in a specific case, simultaneously [5,6]. Clearly, it is difficult to defend a group of attackers bursting into sight at the same time, even though each member is the conventional one in performance. So the simultaneous attack of multiple missiles is a cost-effective and efficient cooperative attack strategy.

A simultaneous attack of a group of missiles against a single common target can be achieved by two ways. The first approach is individual homing, in which a common impact time is commanded to all members in advance, and thereafter each missile tries to home on the target on time independently. The second is cooperative homing, in which the missiles communicate among themselves to synchronize the arrival times. In other words, the missiles with larger times-to-go try to take shortcuts, whereas others with shorter times-to-go take detours to delay the arrival times. The first concept requires determination of a suitable common impact time before homing, but the second needs online data links throughout the engagement.

Despite a number of studies on guidance problems related to time-to-go [7–10], studies on guidance laws to control impact time for a simultaneous attack are rare, except a few recent works by the authors. An impact-time-control guidance law (ITCG) for antiship missiles was developed in [5] and, as an extension of this study, a guidance law to control both impact time and angle (ITACG) was presented in [11]. These individual homing methods are based on optimal control theory, providing analytical closed-loop guidance laws. Herein, the desired impact time is assumed to be prescribed before the homing phase starts. Alternatively, this Note is concerned with a new guidance law based on the second approach, cooperative homing, for a simultaneous attack of multiple missiles.

Proportional navigation (PN) is a well-known homing guidance method in which the rate of turn of the interceptor is made proportional with a navigation ratio N to the rate of turn of the line of sight (LOS) between the interceptor and the target. The navigation constant N is a unitless gain chosen in the range from 3 to 5 [12]. Although PN with $N = 3$ is known to be energy-optimal, an arbitrary $N > 3$ is also optimal if a time-varying weighting function is included into the cost function of the linear quadratic energy-optimal problem [13,14]. In general, the navigation ratio is held fixed. In some cases, however, it can be considered as a control parameter to achieve a desired terminal heading angle [15]. Although PN results in successful intercepts under a wide range of engagement conditions, its control-efficiency is not optimal, in general, especially for the case of maneuvering targets [16]. Augmented proportional navigation, a variant of PN, is useful in cases in which target maneuvers are significant [12]. Biased proportional navigation is also commonly used to compensate for target accelerations and sensor noises or to achieve a desired attitude angle at impact [17]. Even if PN and its variants are already well known and widely used, they are not directly applicable to many-to-one engagements.

This Note proposes a homing guidance law called cooperative proportional navigation (CPN) for many-to-one engagements: CPN has the same structure as conventional PN except that it has a time-varying navigation gain that is adjusted based on the onboard time-to-go and the times-to-go of the other missiles. CPN uses the time-varying navigation gain as a control parameter for reducing the variance of times-on-target of multiple missiles.

This Note begins with the formulation of the homing problem of multiple missiles against a single target, subject to constraints on the impact time. Next, preliminary concepts such as the relative time-to-go error and the variance of times-to-go of multiple missiles are introduced and a new guidance law is proposed. Then the major property of the law is investigated and the characteristics of the law for the case of two missiles are examined in detail. Finally, numerical simulation results illustrate the performances of the proposed law.

II. Problem Statement

Consider the homing guidance geometry of an antiship missile M attacking a stationary target T, as shown in Fig. 1. The target is modeled as being stationary, since the maneuverability and the speed

Received 29 July 2008; revision received 25 August 2009; accepted for publication 4 September 2009.

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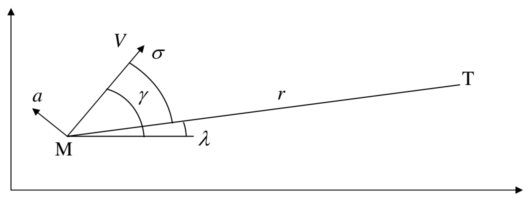

Fig. 1 Guidance geometry on one-to-one engagement scenario.

of surface ships are not comparable with those of antiship missiles of high-subsonic or supersonic speed. The missile speed V is assumed to be constant during the engagement. The missile is controlled by the acceleration command a, which is perpendicular to missile velocity.

Let us suppose that the missile uses well-known PN for homing as

$$a = NV\dot{\lambda} \tag{1}$$

where N denotes the effective navigation constant and $\dot{\lambda}$ is the rate of line-of-sight angle, respectively. Then the flight-path angle is calculated from

$$\dot{\gamma} = a/V \tag{2}$$

or

$$\dot{\gamma} = N\dot{\lambda} \tag{3}$$

The rate of the line of sight can be obtained as

$$\dot{\lambda} = -\frac{V \sin\sigma}{r} \tag{4}$$

where r is the range-to-go and $\sigma = \gamma - \lambda$. Thus, the governing equations in this homing problem can be expressed in terms of r and σ as

$$\dot{r} = -V\cos\sigma \tag{5}$$

$$\dot{\sigma} = -\frac{(N-1)V\sin\sigma}{r} \tag{6}$$

In general, a large N causes sensitivity to noises, whereas a small N results in slow response and degradation of homing performance against an agile target. Thus, the choice of N, which is usually taken between 3 and 5 [12], is based on the designer's experience.

If the missile flies straight along the LOS (that is, $\sigma = 0$), then the time-to-go estimate $\hat{t}_{\mathrm{go},L}$ should be r/V, regardless of N. When σ is not zero, but small, the time-to-go estimate of PN can be approximated as

$$\hat{t}_{\mathrm{go}} = \hat{t}_{\mathrm{go},L}(1+\delta) \tag{7}$$

where

$$\delta = \frac{\sigma^2}{2(2N-1)}$$

represents the increment of the time-to-go due to the initial heading error σ [18] (see Appendix A). Figure 2 shows the time-to-go increments with respect to the navigation constant N. This figure shows that use of time-invariant N may not be an effective strategy for flight-time control. Instead, a time-varying navigation gain of a wide range is required.

Now suppose that m missiles participate in a salvo attack. Figure 3 shows the guidance geometry of the many-to-one engagement scenario. We assume that the speed of each missile is constant but may not be the same as the speed of other missiles. Although each missile has different initial conditions, their common aim is to reach the target at the same time as much as possible.

The problem to be investigated here is to find a time-varying navigation gain $\bar{N}_i$ of PN given as

$$a_i = \bar{N}_i V_i \dot{\lambda}_i, \qquad i = 1, \ldots, m \tag{8}$$

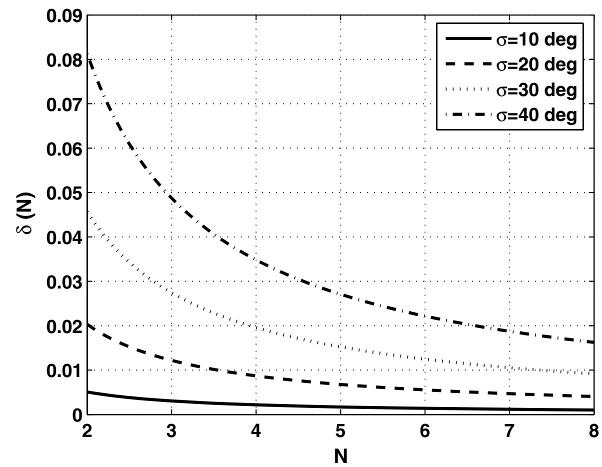

Fig. 2 Time-to-go increments versus *N*.

for which the group of missiles participating in the salvo attack can achieve a simultaneous attack. The subscript i represents the ith missile. Conceptually, the missiles located far from the target should use a high navigation gain to reduce the flight time, whereas those nearer should use a low navigation gain to provide a detour intentionally. Now the time-varying navigation gain to be obtained in the given problem plays a role not only for homing to the target but also for controlling the flight time.

III. Guidance Law for Cooperative Attack

It is known that the PN command given as

$$a = (N_0 + \eta)V\dot{\lambda} \tag{9}$$

minimizes the cost function:

$$J = \frac{1}{2}\int_0^{t_f} \frac{1}{(t_f-\tau)^{\eta}} a^2(\tau)\mathrm{d}\tau \tag{10}$$

where $N_0 = 3$ and $\eta \geq 0$ [19]. As the gain η increases, much greater weight is placed on the control usage as the time-to-go $t_f - t$ goes to zero. Note that the gain η can be used to modify the curvature of the flight trajectory while enforcing a specified final position. Motivated by this fact, a guidance law similar to Eq. (9) will be exploited in this Note.

First, suppose that the time-varying navigation gain $\bar{N}$ of Eq. (8) takes the form given by

$$\bar{N}_i(t) = N(1 - \Omega_i(t)), \qquad i = 1, \ldots, m \tag{11}$$

where N is a navigation constant and $\Omega_i(t)$ is a time-varying gain ratio. Then the problem of determining $\bar{N}_i(t)$ is equivalent to determining $\Omega_i(t)$.

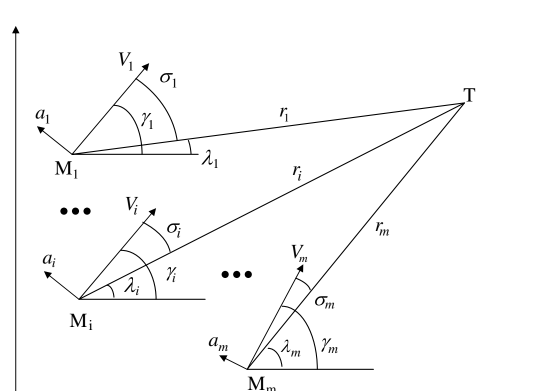

Fig. 3 Guidance geometry on many-to-one engagement scenario.

Next, define the relative time-to-go error of the ith missile as

$$\hat{\varepsilon}_i(t) = \left(\frac{1}{m-1}\sum_{j=1, j\neq i}^{m} \hat{t}_{\mathrm{go},j}(t)\right) - \hat{t}_{\mathrm{go},i}(t) \tag{12}$$

where $\hat{t}_{\mathrm{go},i}(t)$ is defined as

$$\hat{t}_{\mathrm{go},i}(t) \approx \frac{r_i(t)}{V_i}\left(1 + \frac{\sigma_i^2(t)}{2(2N-1)}\right) \tag{13}$$

The relative time-to-go error is the difference between a time-to-go of the ith missile and the mean of times-to-go of the others. Equation (12) can be rearranged as

$$\hat{\varepsilon}_i(t) = \frac{m}{m-1}(\bar{t}_{\mathrm{go}}(t) - \hat{t}_{\mathrm{go},i}(t)) \tag{14}$$

where

$$\bar{t}_{\mathrm{go}}(t) = \frac{1}{m}\sum_{j=1}^{m} \hat{t}_{\mathrm{go},j}(t) \tag{15}$$

which is the mean of $\hat{t}_{\mathrm{go}}s$ of all missiles. Then the variance of times-to-go, $\Sigma^2(t)$, is calculated as

$$\Sigma^2(t) = \frac{1}{m}\sum_{j=1}^{m}(\bar{t}_{\mathrm{go}}(t) - \hat{t}_{\mathrm{go},j}(t))^2 \tag{16}$$

If $\Sigma^2(t) = 0$ at any time, then all missiles can reach the target simultaneously by maintaining the navigation gain fixed at the current value. Hence, the variance can be regarded as a performance indicator for a cooperative simultaneous attack of multiple missiles employing PN with the navigation constant of N. For convenience, this Note calls the variance defined in Eq. (16) the t_{go} variance. It is the central idea of this Note that a simultaneous attack can be achieved if the t_{go} variance can be reduced to zero by adjusting $\Omega_i(t)$.

If the time-varying navigation gain $\bar{N}_i(t)$ of Eq. (11) is applied, the governing equations of the ith missile can be written as from Eqs. (5) and (6):

$$\dot{r}_i(t) = -V_i \cos\sigma_i(t) \tag{17}$$

$$\dot{\sigma}_i(t) = -\frac{(N-1)V_i \sin\sigma_i(t)}{r_i(t)} + \frac{NV_i \sin\sigma_i(t)}{r_i(t)}\Omega_i(t) \tag{18}$$

where the terminal conditions are given by $r_i(t_{f,i}) = 0$.

We propose a homing guidance law, called CPN in this Note, in a very concise form as

$$\Omega_i(t) = Kr_i(t)\hat{\varepsilon}_i(t) \tag{19}$$

Note that $\bar{N}_i(t) \to N$ as $r_i(t) \to 0$ if $\Omega_i(t) \to 0$ as $r_i(t) \to 0$.

Theorem: For a positive gain K, the proposed law of Eq. (19) subject to Eqs. (17) and (18) makes the t_{go} variance $\Sigma^2(t)$ of Eq. (16) decrease during the homing guidance.

Proof: The governing equations of Eqs. (17) and (18) can be expressed as

$$\dot{r}_i(t) = -V_i\left(1 - \frac{\sigma_i^2(t)}{2}\right) \tag{20}$$

$$\dot{\sigma}_i(t) = -\frac{(N-1)V_i\sigma_i(t)}{r_i(t)} + \frac{NV_i\sigma_i(t)}{r_i(t)}\Omega_i(t) \tag{21}$$

under the small angle assumption of $\sigma_i(t)$:

$$\sin\sigma_i(t) = \sigma_i(t) + \mathrm{O}(\sigma_i^3(t)) \tag{22}$$

$$\cos\sigma_i(t) = 1 - \frac{\sigma_i^2(t)}{2} + \mathrm{O}(\sigma_i^4(t)) \tag{23}$$

We get the difference equations from these differential equations as

$$r_i(t+\Delta t) = r_i(t) - V_i\Delta t\left(1 - \frac{\sigma_i^2}{2}\right) \tag{24}$$

$$\sigma_i(t+\Delta t) = \sigma_i(t) - \frac{(N-1)V_i\sigma_i(t)\Delta t}{r_i(t)} + \frac{NV_i\sigma_i(t)\Delta t}{r_i(t)}\Omega_i(t) \tag{25}$$

where t represents the current time step and $t + \Delta t$ is the next step. Neglecting the higher-order terms of Δt, we have

$$\sigma_i^2(t+\Delta t) = \sigma_i^2(t) - 2\sigma_i^2(t)\Delta t\left(\frac{(N-1)V_i}{r_i(t)} - \frac{NV_i}{r_i(t)}\Omega_i(t)\right) \tag{26}$$

Then the time-to-go of the ith missile at the next step can be expressed from Eq. (13) as

$$\hat{t}_{\mathrm{go},i}(t+\Delta t) = \hat{t}_{\mathrm{go},i}(t) - \frac{\Delta t}{2N-1}((2N-1) - N\sigma_i^2(t)\Omega_i(t)) \tag{27}$$

And, from Eq. (15),

$$\bar{t}_{\mathrm{go}}(t+\Delta t) = \bar{t}_{\mathrm{go}}(t) - \frac{\Delta t}{2N-1}\left((2N-1) - \frac{N}{m}\sum_{j=1}^{m}\sigma_j^2(t)\Omega_j(t)\right) \tag{28}$$

The variance at the next step can be expressed as

$$\Sigma^2(t+\Delta t) = \frac{1}{m}\sum_{j=1}^{m}(\bar{t}_{\mathrm{go}}(t+\Delta t) - \hat{t}_{\mathrm{go},j}(t+\Delta t))^2 \tag{29}$$

Substituting Eqs. (27) and (28) into Eq. (29) yields

$$\Sigma^2(t+\Delta t) = \Sigma^2(t) - \frac{2N\Delta t}{m(2N-1)}\sum_{j=1}^{m}(\bar{t}_{\mathrm{go}}(t) - \hat{t}_{\mathrm{go},j}(t))\sigma_j^2(t)\Omega_j(t) \tag{30}$$

Note that if $\Omega_j(t) = 0$ for all j, then $\Sigma^2(t+\Delta t) = \Sigma^2(t)$ (i.e., t_{go} variance does not decrease). From Eqs. (14) and (19), we have

$$\Omega_j(t) = \frac{Km}{m-1}r_j(t)(\bar{t}_{\mathrm{go}}(t) - \hat{t}_{\mathrm{go},j}(t)) \tag{31}$$

Substituting Eq. (31) into Eq. (30) yields

$$\Sigma^2(t+\Delta t) = \Sigma^2(t) - K\alpha(t)\Delta t \tag{32}$$

where

$$\alpha(t) = \frac{2N}{(2N-1)(m-1)}\sum_{j=1}^{m} r_j(t)\sigma_j^2(t)(\bar{t}_{\mathrm{go}}(t) - \hat{t}_{\mathrm{go},j}(t))^2 \tag{33}$$

Note that $\alpha(t)$ of Eq. (33) is always nonnegative. So the variance of Eq. (16) decreases for a properly chosen K. Especially, it decreases monotonically as long as the summation of the right-hand side (RHS) of (33) is not zero. QED

Note that the summation of RHS of Eq. (33) is zero when all of the ranges-to-go go to zero or when the variance of the times-to-go goes to zero. But it also occurs undesirably when all of the heading errors go to zero, although in a real situation, it may hardly happen during engagement except at terminal stage.

The proposed law achieves two objectives: a reliable target intercept by using a typical navigation constant N near the target and reduction of the t_{go} variance via trajectory reshaping based on the proposed gain adjustment. The trajectory reshaping is effective when the missile is far away from the target, whereas target intercept is most important when it is near the target. Note that CPN becomes conventional PN as the range-to-go decreases.

CPN has a flexibility to allow the missiles to fly at different speeds for a cooperative simultaneous attack. And CPN requires no additional information, compared to PN, for implementation except onboard range-to-go and time-to-go calculations of all of the missiles.

Let us investigate the properties of the proposed law for the case of two missiles ($m = 2$). The relative time-to-go error of Eq. (14) is given by

$$\hat{\varepsilon}_1(t) = \hat{t}_{\mathrm{go},2}(t) - \hat{t}_{\mathrm{go},1}(t) \quad \text{and} \quad \hat{\varepsilon}_2(t) = -\hat{\varepsilon}_1(t) \tag{34}$$

In the case of $m = 2$, the relative time-to-go error is the difference between times-to-go of the two missiles. The gain ratio of Eq. (19) is given as

Table 1 Engagement scenario for two missiles

Parameters	Missile 1	Missile 2
Initial position, km	(1.18, 2.08)	(0, 0)
Initial heading, deg	−20	−5
Velocity, m/s	290	300
Target position, km	(13, 0)	(13, 0)

$$\Omega_1(t) = Kr_1(t)\hat{\varepsilon}_1(t) \quad \text{and} \quad \Omega_2(t) = -Kr_2(t)\hat{\varepsilon}_1(t) \tag{35}$$

Thus, rearranging Eq. (32) yields

$$(\hat{t}_{\text{go},1}(t+\Delta t) - \hat{t}_{\text{go},2}(t+\Delta t))^2 = (\hat{t}_{\text{go},1}(t) - \hat{t}_{\text{go},2}(t))^2(1 - K\beta(t)\Delta t) \tag{36}$$

or

$$\frac{\hat{\varepsilon}_1^2(t+\Delta t) - \hat{\varepsilon}_1^2(t)}{\Delta t} = -K\beta(t)\hat{\varepsilon}_1^2(t) \tag{37}$$

where

$$\beta(t) = \frac{2N}{(2N-1)}(r_1(t)\sigma_1^2(t) + r_2(t)\sigma_2^2(t)) \tag{38}$$

Taking the limit as $\Delta t \to 0$ gives the solution of Eq. (36) as

$$\hat{\varepsilon}_1^2(t) = \hat{\varepsilon}_1^2(0)e^{-K\int_0^t \beta(\tau)\mathrm{d}\tau} \tag{39}$$

The result shows that the law explicitly reduces the difference of times-to-go of two missiles, since $\beta(t)$ is nonnegative.

IV. Numerical Simulation

Let us consider an engagement scenario in which two missiles with constant speeds of 299 and 300 m/s, respectively, attack a single stationary ship target. Initial ranges-to-go are 13 and 12 km, respectively, as shown in Table 1.

In case of PN with $N = 3$, the impact times are observed in simulation to be about 45.4 and 40.0 s, respectively, as shown in Fig. 4. There exists about 5.4 s of a difference in flight time. To apply the proposed law to this scenario, the gain K is properly chosen as $K = 40/(\bar{r}_0\bar{t}_{\text{go}0})$, where the subscript 0 denotes an initial time and the superscript bar represents the mean value. CPN with $N = 3$ reduces the discrepancy of impact times within 0.1 s around the impact time of 44.9 s. It results from cooperative interactions between two missiles for reducing the difference of flight times. The flight trajectories of two missiles by CPN are depicted in comparison with the trajectories by PN in Fig. 4. The difference of times-to-go and the standard deviation of the difference of times-to-go are reduced by the law, as can be seen in Figs. 5 and 6, respectively. It is obvious from Fig. 7 that the gain ratios of two missiles are nullified during the homing guidance in this scenario and thus CPN gradually becomes conventional PN with a navigation constant of $N = 3$. It means that, except an initial stage for a flight-time control, a stable intercept can be achieved in the level of PN with a time-invariant $N = 3$. Figure 8 shows the time histories of acceleration commands of two missiles. Compared with PN, much more control efforts are required for CPN.

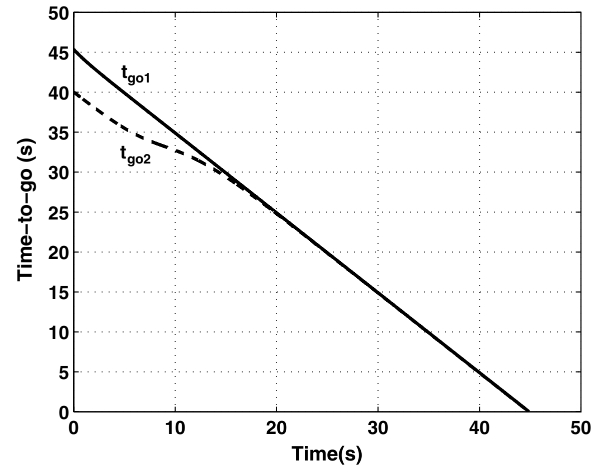

Fig. 5 Time-to-go histories.

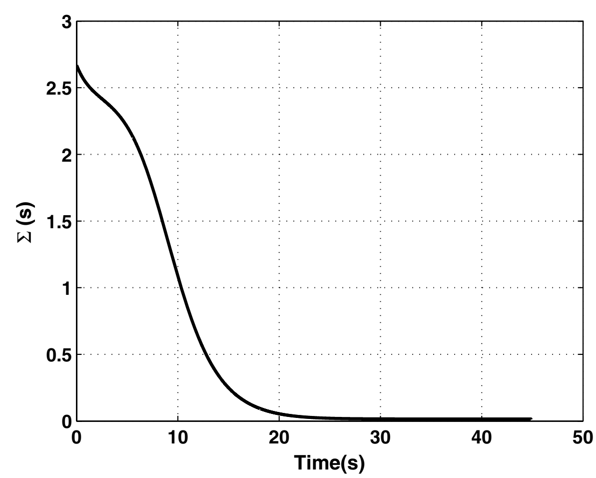

Fig. 6 Standard deviation of difference of times-to-go for two missiles.

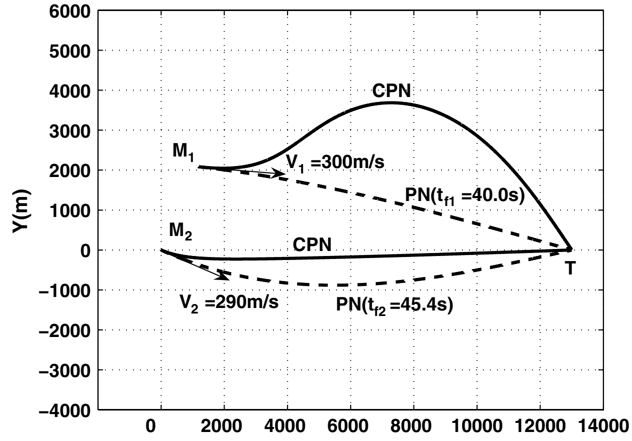

Fig. 4 Flight trajectories for two missiles.

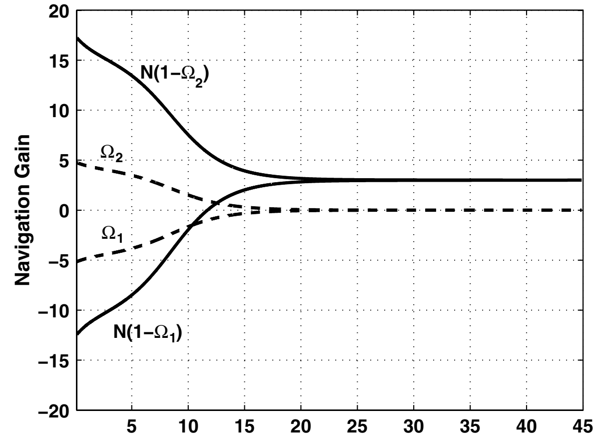

Fig. 7 Navigation gains for two missiles.

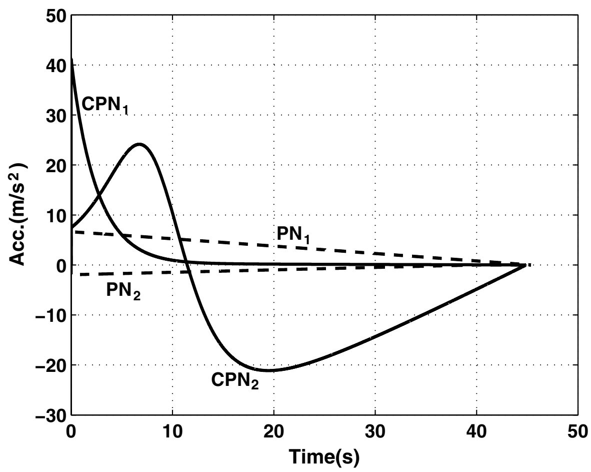

Fig. 8 Acceleration commands for two missiles.

V. Conclusions

In this Note, the homing guidance problem for a cooperative simultaneous attack of multiple missiles against their common target is discussed. Introducing a new concept of the t_{go} variance of multiple missiles, we propose the cooperative proportional navigation guidance law, which can achieve a simultaneous attack by decreasing the t_{go} variance cooperatively till the intercept. The law shows some good features. First, the law does not require that the desired common impact time be commanded to all members in advance. Second, the law has a feasible and practical structure: for implementation, it requires only two more additional items of information than with conventional PN: the range-to-go of own missile and times-to-go of the others. Finally, the law has the flexibility to allow missiles with different speeds to carry out a cooperative simultaneous attack. Thus, the proposed law can be easily applicable to many-to-one engagement scenarios of typical antiship missiles. Simulation results clearly demonstrate the feasibility and the homing performance of the proposed guidance law.

Appendix A: Derivation of Time-to-Go Approximation of PN

To find the time-to-go expression associated with an initial range-to-go and a heading angle to LOS, we consider the inertial coordinate system shown in Fig. A1. The origin of the coordinate system lies on the initial position of missile. Here, the x axis is directed to the target position and the y axis is normal to x axis. The governing equations are given by

$$\dot{x} = V\cos\gamma \qquad \dot{y} = V\sin\gamma \qquad \dot{\gamma} = a/V \tag{A1}$$

where the dot denotes the differentiation with respect to time. Under the assumptions of small γ angle, constant speed V, and acceleration command a normal to velocity, substituting x/V with time t leads to

$$y' = \gamma \qquad \gamma' = a/V^2 \tag{A2}$$

where the prime represents the derivative with respect to the downrange x. Since the line-of-sight angle λ can be approximated as $-y/(x_f - x)$, the PN command a is

$$a(x) = NV\lambda' = -\tfrac{NV^2}{(x_f-x)^2}y - \tfrac{NV^2}{(x_f-x)}y' \tag{A3}$$

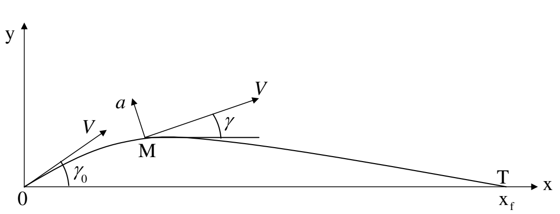

Fig. A1 Guidance geometry for time-to-go calculation.

Substitute Eq. (A3) into Eq. (A2) to get

$$y'' + \tfrac{N}{x_f-x}y' + \tfrac{N}{(x_f-x)^2}y = 0 \tag{A4}$$

where the initial conditions are given by $y(0) = 0$ and $y'(0) = \gamma_0$. The solution of this Cauchy equation can be easily obtained as

$$y(x) = \tfrac{\gamma_0}{N-1}(x_f - x)\left(1 - \left(1 - \tfrac{x}{x_f}\right)^{N-1}\right) \tag{44}$$

Also differentiating Eq. (A5) with respect to x yields

$$\gamma(x) = -\tfrac{\gamma_0}{N-1}\left(1 - N\left(1 - \tfrac{x}{x_f}\right)^{N-1}\right) \tag{A5}$$

The length of the trajectory of Eq. (A5), s, is given by

$$s = Vt_f = \int_0^{x_f}\sqrt{1+y'^2}\,\mathrm{d}x \tag{A7}$$

If $y'(=\gamma)$ is assumed to be small, Eq. (A7) can be approximated as

$$Vt_f \approx \int_0^{x_f}\left(1 + \tfrac{1}{2}y'^2\right)\mathrm{d}x = x_f\left(1 + \tfrac{\gamma_0^2}{2(2N-1)}\right) \tag{A8}$$

Thus,

$$t_f \approx \tfrac{x_f}{V}\left(1 + \tfrac{\gamma_0^2}{2(2N-1)}\right) \tag{A9}$$

where t_f can be regarded as a time-to-go estimation at initial time. One can always set the inertial coordinate system at initial time as some specific system with the x axis being consistent with the current LOS. As a result, an initial heading angle γ_0 coincides with σ in Fig. 1. Thus, the time-to-go approximation of PN in terms of two states r and σ can be expressed as

$$\hat{t}_{go} = \tfrac{r}{V}\left(1 + \tfrac{\sigma^2}{2(2N-1)}\right) \tag{A10}$$

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